

Template for XCPC

SUNCHAOYI

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1 Graph

1.1 Diameter of a Tree

```
template <typename T>
class Diameter
{
    int n,p;
    vector <T> dis;
    vector <vector <pair <int,T>>> ve;
    void dfs (int u,int fa)
    {
        for (const auto& [v,w] : ve[u])
        {
            if (v == fa) continue;
            dis[v] = dis[u] + w;
            if (dis[v] > dis[p]) p = v;
            dfs (v,u);
        }
    }
public:
    Diameter (int n) : n(n),ve(n + 1) {}
    void add (int u,int v,T w = 1) {ve[u].push_back ({v,w});ve[v].push_back ({u,w});}
    T calc ()
    {
        dis.assign (n + 1,0);
        p = 1;dfs (1,0);
        dis[p] = 0;dfs (p,0);
        return dis[p];
    }
};
```

1.2 Centroid of a Tree

```
class Centroid
{
    int n;
    vector <int> sz,w,cen;
    vector <vector <int>> ve;
    void dfs (int u,int fa)
    {
        sz[u] = 1;w[u] = 0;
        for (auto v : ve[u])
        {
            if (v == fa) continue;
            dfs (v,u);
            sz[u] += sz[v];
            w[u] = max (w[u],sz[v]);
        }
        w[u] = max (w[u],n - sz[u]);
        if (w[u] <= n / 2) cen.push_back (u);
    }
public:
    Centroid (int n) : n(n),ve (n + 1),sz (n + 1),w (n + 1) {}
    void add (int u,int v) {ve[u].push_back (v);ve[v].push_back (u);}
    auto calc ()
    {
        cen.clear ();
        dfs (1, 0);
        sort (cen.begin (),cen.end ());
        return cen;
    }
};
```

```
};
```

1.3 Minimum Spanning Tree

1.3.1 Prim

```
template <typename T>
class MST
{
    int n;T ans;
    vector <int> vis;vector <vector <int>> g;vector <T> dis;
public :
    MST (int n) : n (n),vis (n + 1,0),g (n + 1,vector <int> (n + 1,INF)),dis (n +
        1,INF) {dis[1] = ans = 0;vis[1] = 1;}
    void add (int u,int v,T w) {g[u][v] = g[v][u] = w;}
    T calc ()
    {
        for (int i = 2;i <= n;++i) dis[i] = g[1][i];
        for (int i = 1;i < n;++i)
        {
            int k = 0;
            for (int j = 1;j <= n;++j)
                if (!vis[j] && dis[j] < dis[k]) k = j;
            vis[k] = 1;
            ans += dis[k];
            for (int j = 1;j <= n;++j)
                if (!vis[j] && g[k][j] < dis[j]) dis[j] = g[k][j];
        }
        return ans;
    }
};
```

1.3.2 Kruskal

```
template <typename T>
class MST
{
    int n,m,e_cnt,cnt;T ans;
    struct node {int u,v;T w;};
    vector <int> fa;vector <node> g;
public:
    MST (int n,int m) : n (n),m(m),fa (n + 1,0),g (m + 1) {cnt = e_cnt = ans =
        0;}
    void add (int u,int v,int w) {g[++e_cnt].u = u,g[e_cnt].v = v,g[e_cnt].w = w
        ;}
    int getfa (int u) {return fa[u] == u ? u : fa[u] = getfa (fa[u]);}
    T calc ()
    {
        sort (g.begin (),g.end (),[] (auto &x,auto &y) {return x.w < y.w;});
        for (int i = 1;i <= n;++i) fa[i] = i;
        for (int i = 1;cnt < n && i <= m;++i)
        {
            int dx = getfa (g[i].u),dy = getfa (g[i].v);
            if (dx == dy) continue;
            ans += g[i].w;fa[dx] = dy;++cnt;
        }
        return ans;
    }
};
```

1.4 LCA

```
class LCA
{
```

```

static constexpr int lg = 20;
int n;
vector<int> dep;
vector<vector<int>> f,ve;
public:
LCA (int n) : n (n),ve (n + 1),dep (n + 1),f (n + 1,vector<int> (lg + 1,0))
{}
void add (int u,int v) {ve[u].push_back (v);ve[v].push_back (u);}
void pre (int u,int fa)
{
    f[u][0] = fa;dep[u] = dep[fa] + 1;
    for (int i = 0;i < lg;++i) f[u][i + 1] = f[f[u][i]][i];
    for (auto v : ve[u])
        if (v != fa) pre (v,u);
}
int query (int u,int v)
{
    if (dep[u] < dep[v]) swap (u,v);
    for (int i = lg;~i;--i)
    {
        if (dep[f[u][i]] >= dep[v]) u = f[u][i];
        if (u == v) return u;
    }
    for (int i = lg;~i;--i)
        if (f[u][i] != f[v][i]) u = f[u][i],v = f[v][i];
    return f[u][0];
}
};

```

1.5 Topological Sorting

```

class Topo
{
    int n;
    vector<int> deg;
    vector<vector<int>> ve;

public:
Topo (int n) : n (n),ve (n + 1),deg (n + 1,0) {}
void add (int u,int v)
{
    ve[u].push_back (v);
    ++deg[v];
}
vector<int> calc ()
{
    queue<int> q;
    vector<int> lst;
    for (int i = 1;i <= n;++i)
        if (!deg[i]) q.push (i);
    while (!q.empty ())
    {
        int u = q.front ();q.pop ();
        lst.push_back (u);
        for (auto v : ve[u])
        {
            --deg[v];
            if (!deg[v]) q.push (v);
        }
    }
    return lst;
}
};

```

1.6 Shortest Path

1.6.1 Dijkstra

```
class dijkstra
{
    int n,m,cnt;
    vector <int> head,to,nxt,vis;
    vector <ll> dis,val;
public:
    dijkstra (int n,int m) :
        n (n),m (m),vis (n + 1,0),head (n + 1,0),dis (n + 1,1e18),
        to (2 * m + 1,0),nxt (2 * m + 1,0),val (2 * m + 1,0) {cnt = 0;}
    void add (int u,int v,int w)
    {
        to[++cnt] = v;val[cnt] = w;nxt[cnt] = head[u];head[u] = cnt;
        to[++cnt] = u;val[cnt] = w;nxt[cnt] = head[v];head[v] = cnt;
    }
    auto calc (int s)
    {
        priority_queue <pair <ll,int>> q;
        for (int i = 1;i <= n;++i) vis[i] = 0,dis[i] = INF;
        q.push ({0,s});
        dis[s] = 0;
        while (!q.empty ())
        {
            int u = q.top ().second;q.pop ();
            if (vis[u]) continue;
            vis[u] = 1;
            for (int i = head[u];i;i = nxt[i])
            {
                int v = to[i];
                if (dis[v] > dis[u] + val[i])
                {
                    dis[v] = dis[u] + val[i];
                    q.push ({-dis[v],v});
                }
            }
        }
        return dis;
    }
};
```

1.6.2 SPFA

```
class SPFA
{
    int n,m,cnt;
    vector <int> head,to,nxt,val,vis,times;
    vector <ll> dis;
public:
    SPFA (int n,int m) :
        n (n),m (m),times (n + 1,0),vis (n + 1,0),head (n + 1,0),dis (n + 1,INF),
        to (2 * m + 1,0),nxt (2 * m + 1,0),val (2 * m + 1,0) {cnt = 0;}
    void add (int u,int v,int w)
    {
        to[++cnt] = v;val[cnt] = w;nxt[cnt] = head[u];head[u] = cnt;
        to[++cnt] = u;val[cnt] = w;nxt[cnt] = head[v];head[v] = cnt;
    }
    auto calc (int s)
    {
        queue <int> q;
        for (int i = 1;i <= n;++i) vis[i] = 0,dis[i] = INF;
        dis[s] = 0,vis[s] = 1;q.push (s);
    }
};
```

```

while (!q.empty())
{
    int u = q.front ();
    q.pop (),vis[u] = 0;
    for (int i = head[u];i;i = nxt[i])
    {
        int v = to[i];
        if (dis[v] > dis[u] + val[i])
        {
            dis[v] = dis[u] + val[i];
            times[v] = times[u] + 1;
            if (times[v] >= n) return {-1};//Negative Cycle
            if (!vis[v]) q.push (v),vis[v] = 1;
        }
    }
}
return dis;
}
};

```

1.7 DCC

```

class DCC
{
public:
    struct Edge {int to,nxt;};
    vector <Edge> G;
    vector <int> head,dfn,low,col;
    vector <bool> cut;
    stack <int> stk;
    int cnt,cct,n;
    void init (int nn = 0)
    {
        G.assign (2,{0,0});
        n = nn;head.assign (n + 1,0);
    }
    void dfs (int u,int f = -1)
    {
        dfn[u] = low[u] = ++cnt;
        int ch = 0;
        for (int i = head[u];i;i = G[i].nxt)
        if (i != f)
        {
            int v = G[i].to;
            if (dfn[v] < dfn[u]) stk.push (i >> 1);
            if (!dfn[v])
            {
                dfs (v,i ^ 1);++ch;
                low[u] = min (low[u],low[v]);
                if (low[v] >= dfn[u])
                {
                    int I = 0;
                    ++cct;cut[u] = true;
                    do
                    {
                        assert (!stk.empty ());
                        I = stk.top ();stk.pop ();
                        col[I] = cct;
                    }
                    while (I != (i >> 1));
                }
            }
        }
        else if (dfn[v] < low[u]) low[u] = dfn[v];
    }
}

```

```

        if (f == -1 && ch == 1) cut[u] = false;
    }
    void tarjan ()
    {
        cnt = cct = 0;
        col.assign (G.size () >> 1,0);
        dfn.assign (n + 1,0);
        low.assign (n + 1,0);
        cut.assign (n + 1,false);
        while (!stk.empty ()) stk.pop ();
        for (int i = 1;i <= n;++i) if (!dfn[i]) dfs (i);
    }
    void insert (int u,int v)
    {
        G.push_back ({v,head[u]});head[u] = G.size () - 1;
        G.push_back ({u,head[v]});head[v] = G.size () - 1;
    }
    void insert (vector <vector <int>> &G1)
    {
        for (unsigned u = 1;u < G1.size ();++u)
            for (int v : G1[u]) insert (u,v);
    }
    bool operator [] (const int &x) const {return cut[x];}
};

```

1.8 Tarjan

```

class Tarjan
{
    int n,m,cnt,times,scc_cnt;
    vector <int> head,to,nxt,low,scc,dfn;
    stack <int> s;
    void tarjan (int u)
    {
        low[u] = dfn[u] = ++times;
        s.push (u);
        for (int i = head[u];i;i = nxt[i])
        {
            int v = to[i];
            if (!dfn[v])
            {
                tarjan (v);
                low[u] = min (low[u],low[v]);
            }
            else if (!scc[v]) low[u] = min (low[u],dfn[v]);
        }
        if (low[u] == dfn[u])
        {
            ++scc_cnt;
            while (1)
            {
                int x = s.top ();s.pop ();
                scc[x] = scc_cnt;
                if (x == u) break;
            }
        }
    }
public:
    Tarjan (int n,int m) :
        n (n),m (m),head (n + 2,0),low (n + 1,0),dfn (n + 1,0),scc (n + 1,0),
        to (2 * m + 2,0),nxt (2 * m + 2,0) {cnt = 1;times = scc_cnt = 0;}
    void add (int u,int v) {to[++cnt] = v;nxt[cnt] = head[u];head[u] = cnt;}
    auto calc ()
    {

```

```

        for (int i = 1; i <= n; ++i)
            if (!dfn[i]) tarjan (i);
        return scc;
    }
};

```

1.9 Bipartite Graph Matchings

```

class Matching
{
    int l,r;//the number of left/right side points
    vector <vector <int>> ve;
    vector <int> vis,op;
    bool dfs (int u)
    {
        for (auto v : ve[u])
        {
            if (vis[v]) continue;
            vis[v] = 1;
            if (!op[v] || dfs (op[v])) {op[v] = u;return true;}
        }
        return false;
    }
public:
    Matching (int l,int r) : l (l),r (r),vis (r + 1,0),op (r + 1,0),ve (l + 1) {}
    void add (int u,int v) {ve[u].push_back (v);}
    int calc ()
    {
        int ans = 0;
        for (int i = 1;i <= l;++i)
        {
            vis.assign (r + 1,0);
            if (dfs (i)) ++ans;
        }
        return ans;
    }
};

```

1.10 Flow

1.10.1 Edmonds-Karp

```

template <typename T>
class EK
{
    int n,m,s,t,cnt;
    vector <int> head,to,nxt,vis,pre,edge;
    vector <T> val;
    bool bfs ()
    {
        queue <T> q;
        for (int i = 1;i <= n;++i) vis[i] = 0,pre[i] = edge[i] = -1;
        vis[s] = 1;q.push (s);
        while (!q.empty ())
        {
            int u = q.front ();q.pop ();
            for (int i = head[u];i;i = nxt[i])
            {
                int v = to[i];
                if (!vis[v] && val[i])
                {
                    pre[v] = u;edge[v] = i;vis[v] = 1;
                    q.push (v);
                    if (v == t) return 1;
                }
            }
        }
    }
};

```

```

    }
    }
    return 0;
}
public :
EK (int n,int m,int s,int t) :
    n (n),m (m),s (s),t (t),
    vis (n + 1,0),head (n + 1,0),pre (n + 1,-1),edge (n + 1,-1),
    to (m + 1,0),nxt (m + 1,0),val (m + 1,0) {cnt = 1;}
void add (int u,int v,T w)
{
    to[++cnt] = v;val[cnt] = w;nxt[cnt] = head[u];head[u] = cnt;
    to[++cnt] = u;val[cnt] = 0;nxt[cnt] = head[v];head[v] = cnt;
}
T calc ()
{
    T ans = 0;
    while (bfs ())
    {
        T mn = INF;
        for (int i = t;i != s;i = pre[i]) mn = min (mn,val[edge[i]]);
        for (int i = t;i != s;i = pre[i]) val[edge[i]] -= mn,val[edge[i]] ^ 1)
            += mn;
        ans += mn;
    }
    return ans;
}
};
};

```

1.10.2 Dinic

```

template <typename T>
class Dinic
{
    int n,m,s,t,cnt;
    vector <int> head,to,nxt,cur,dep;
    vector <T> val;
    int bfs ()
    {
        for (int i = 0;i <= n;++i) dep[i] = 0,cur[i] = head[i];
        queue <int> q;
        q.push (s),dep[s] = 1;
        while (!q.empty ())
        {
            int u = q.front ();q.pop ();
            for (int i = head[u];i;i = nxt[i])
            {
                int v = to[i];
                if (val[i] && !dep[v]) q.push (v),dep[v] = dep[u] + 1;
            }
        }
        return dep[t];
    }
    T dfs (int u,int t,T flow)
    {
        if (u == t) return flow;
        T ans = 0;
        for (int &i = cur[u];i && ans < flow;i = nxt[i])
        {
            int v = to[i];
            if (val[i] && dep[v] == dep[u] + 1)
            {
                int x = dfs (v,t,min (val[i], flow - ans));
            }
        }
    }
};

```

```

        if (x) val[i] -= x, val[i ^ 1] += x, ans += x;
    }
}
if (ans < flow) dep[u] = -1;
return ans;
}
public :
Dinic (int n, int m, int s, int t) :
    n (n), m (m), s (s), t (t),
    head (n + 1, 0), cur (n + 1, 0), dep (n + 1, 0),
    to (m + 1, 0), nxt (m + 1, 0), val (m + 1, 0) {cnt = 1;}
void add (int u, int v, T w)
{
    to[++cnt] = v; val[cnt] = w; nxt[cnt] = head[u]; head[u] = cnt;
    to[++cnt] = u; val[cnt] = 0; nxt[cnt] = head[v]; head[v] = cnt;
}
T calc ()
{
    T ans = 0;
    while (bfs ())
    {
        T x;
        while ((x = dfs (s, t, INF))) ans += x;
    }
    return ans;
}
};
};

```

2 Data Structure

2.1 Segment Tree

2.2 Fenwick Tree

2.3 Heavy-Light Decomposition

2.4 Splay Tree

2.5 Cartesian Tree

```
//heap-order on values and BST-order on keys
stack <int> s;
vector <int> L (n + 1,0),R (n + 1,0);
for (int i = 1;i <= n;++i)
{
    int la = -1;
    while (!s.empty () && a[s.top ()] > a[i]) la = s.top (),s.pop ();
    if (!s.empty ()) R[s.top ()] = i;
    if (la != -1) L[i] = la;
    s.push (i);
}
```

2.6 Sparse Table

```
vector <int> lg (n + 1,-1);
vector <vector <int>> f (20,vector <int> (n + 1,INF));
for (int i = 1;i <= n;++i) lg[i] = lg[i >> 1] + 1;
for (int i = 1;i <= n;++i) f[0][i] = a[i];
for (int j = 1;j < 20;++j)
    for (int i = 1;i + (1 << j) - 1 <= n;++i) f[j][i] = min (f[j - 1][i],f[j - 1][i + (1 << (j - 1))]);
auto query = [&] (int l,int r) -> int {return min (f[lg[r - l + 1]][l],f[lg[r - l + 1]][r - (1 << lg[r - l + 1]) + 1]);};
```

2.7 Persistent Data Structure

2.8 Linear-Basis

```
template <typename T>
class Basis
{
    int n,lg;
    vector <T> p,ex,nw;vector <vector <int>> id;
public :
    Basis (int n,int lg) : n (n),lg (lg),p (lg + 1,0),nw (lg + 1,0),id (lg + 1)
    {}
    void modify (T x,int idx)
    {
        vector <int> cur (n + 1,0);cur[idx] = 1;
        for (int i = lg;~i;--i)
        {
            if (!((1ll << i) & x)) continue;
            if (!p[i]) {p[i] = x;id[i] = cur;break;}
            else
            {
                x ^= p[i];
                for (int j = 1;j <= n;++j) cur[j] ^= id[i][j];
            }
        }
    }
    T get_max ()
    {}
};
```

```

    T ans = 0;
    for (int i = lg; ~i; --i)
        if ((ans ^ p[i]) > ans) ans ^= p[i];
    return ans;
}
T get_min ()
{
    T ans = 0; int cnt = 0;
    for (int i = 0; i <= lg; ++i) cnt += p[i] > 0;
    if (cnt < n) return 0;
    for (int i = 0; i <= lg; ++i)
        if (p[i] > 0) return p[i];
}
void change ()
{
    for (int i = 0; i <= lg; ++i) nw[i] = p[i];
    for (int i = 0; i <= lg; ++i)
        for (int j = i - 1; ~j; --j)
            if (nw[i] >> j & 1) nw[i] ^= nw[j];
    for (int i = 0; i <= lg; ++i)
        if (nw[i]) ex.push_back (i);
}
T get_min_k (T k)
{
    int sz = (int)ex.size ();
    if (sz < n) --k; // element 0
    if (k > (1ll << sz) - 1) return -1;
    T ans = 0;
    for (int j = 0; j < sz; ++j)
        if (k >> j & 1) ans ^= nw[ex[j]];
    return ans;
}
bool find (T x)
{
    for (int i = lg; ~i; --i)
    {
        if (!(x >> i & 1)) continue;
        if (!p[i]) return false;
        x ^= p[i];
    }
    return true;
}
vector <int> getid (T x) // if find (x) == 1
{
    vector <int> res (n + 1, 0);
    for (int i = lg; ~i; --i)
    {
        if (!(x >> i & 1)) continue;
        x ^= p[i];
        for (int j = 1; j <= n; ++j) res[j] ^= id[i][j];
    }
    return res;
}
};

```

3 Math

3.1 Subset Enumeration

```
for (int S = 1; S < (1 << n); ++S)
    for (int S0 = S; S0; S0 = (S0 - 1) & S)
```

3.2 Elementary Arithmetic Bucket

```
template <typename T,int MOD = 1000000007>
class Z
{
    T x;
    Z <T,MOD> qpow (Z x,T y)
    {
        Z <T,MOD> res (1);
        while (y)
        {
            if (y & 1) res *= x;
            x *= x;
            y >>= 1;
        }
        return res;
    }
public :
    Z () : x(0) {}
    Z (T x) : x (x) {}
    T get () {return x;}
    Z & operator = (T o) {x = o % MOD;return *this;}
    Z & operator += (Z o) {x = (x + o.x + MOD) % MOD;return *this;}
    Z & operator -= (Z o) {x = (x - o.x + MOD) % MOD;return *this;}
    Z & operator *= (Z o) {x = 1ll * o.x * x % MOD;return *this;}
    Z & operator ^= (Z o) {x = qpow (x,o.x).x;return *this;}
    Z & operator /= (Z o) {*this *= o ^ (MOD - 2);return *this;}
    Z & operator %= (Z o) {x = x % o.x;return *this;}
    friend Z operator + (Z x,Z y) {return x += y;}
    friend Z operator - (Z x,Z y) {return x -= y;}
    friend Z operator * (Z x,Z y) {return x *= y;}
    friend Z operator / (Z x,Z y) {return x /= y;}
    friend Z operator ^ (Z x,Z y) {return x ^= y;}
    friend Z operator % (Z x,Z y) {return x %= y;}
    friend bool operator == (Z x,Z y) {return x.x == y.x;}
    friend bool operator != (Z x,Z y) {return x.x != y.x;}
    friend istream& operator >> (istream &is,Z& o) {T val;is >> val;o = Z (val);
        return is;}
    friend ostream& operator << (ostream &os,const Z &z) {return os << z.x;}
};
```

3.3 Fraction

```
template <typename T>
class Frac
{
    T p,q; // p / q
    void reduce ()
    {
        assert (q != 0);
        T g = gcd (p,q);
        p /= g,q /= g;
        if (q < 0) p = -p,q = -q;
    }
public :
    Frac (T p = 0, T q = 1) : p (p),q (q) {assert (q != 0);reduce ();}
```

```

T get_p () {return p;}
T get_q () {return q;}
double get_val () {return 1.0 * p / q;}
Frac operator + (const Frac &o) {return Frac (p * o.q + o.p * q,q * o.q);}
Frac operator - (const Frac &o) {return Frac (p * o.q - o.p * q,q * o.q);}
Frac operator * (const Frac &o) {return Frac (p * o.p,q * o.q);}
Frac operator / (const Frac &o) {return Frac (p * o.q,q * o.p);}
Frac& operator += (const Frac &o) {*this = *this + o;return *this;}
Frac& operator -= (const Frac &o) {*this = *this - o;return *this;}
Frac& operator *= (const Frac &o) {*this = *this * o;return *this;}
Frac& operator /= (const Frac &o) {*this = *this / o;return *this;}
Frac operator -() {return Frac (-p,q);}
bool operator >= (const Frac &o) {return p * o.q >= o.p * q;}
bool operator > (const Frac &o) {return p * o.q > o.p * q;}
bool operator == (const Frac &o) {return p * o.q == o.p * q;}
bool operator <= (const Frac &o) {return !(*this > o);}
bool operator < (const Frac &o) {return !(*this >= o);}
bool operator != (const Frac &o) {return !(*this == o);}
};

```

3.4 Combination

```

template <typename T,int MOD = 1000000007>
class COM
{
    int n;
    vector <Z <T,MOD>> fac,inv;
    public :
    COM (int n) : n (n),fac (2 * n + 2),inv (2 * n + 2)
    {
        fac[0] = inv[0] = inv[1] = 1;
        for (int i = 1;i <= 2 * n;++i) fac[i] = fac[i - 1] * i;
        inv[2 * n] = fac[2 * n] ^ (MOD - 2);
        for (int i = 2 * n - 1;i > 0;--i) inv[i] = inv[i + 1] * (i + 1);
    }
    Z <T,MOD> f (int x) {return fac[x];}
    Z <T,MOD> inv_f (int x) {return inv[x];}
    Z <T,MOD> comb (int x,int y)
    {
        if (y < 0 || y > x) return 0;
        else return fac[x] * inv[x - y] * inv[y];
    }
    Z <T,MOD> arr (int x,int y)
    {
        if (y > x) return 0;
        else return fac[x] * inv[x - y];
    }
    Z <T,MOD> catalan (int x) {return comb (2 * x,x) / (x + 1);}
};

```

3.5 The Sieve of Primes & Euler's Totient Function & Möbius Function

```

class Prime
{
    int mx;
    vector <int> p,phi,mu;
    void pre ()
    {
        vector <int> fl (mx + 1,0);
        phi[1] = mu[1] = 1;
        for (int i = 2;i <= mx;++i)
        {
            if (!fl[i]) p.push_back (i),phi[i] = i - 1,mu[i] = -1;
            for (auto v : p)

```

```

        {
            if (i * v > mx) break;
            fl[i * v] = 1;
            if (i % v == 0) {phi[i * v] = phi[i] * v, mu[i * v] = 0; break;}
            phi[i * v] = phi[i] * (v - 1); mu[i * v] = -mu[i];
        }
    }
}
public :
Prime (int mx) : mx (mx), phi (mx + 1, 0), mu (mx + 1, 0) {pre ();}
auto get_prime () {return p;}
auto get_phi () {return phi;}
auto get_mu () {return mu;}
};

```

3.6 Exgcd

```

ll exgcd (ll a, ll b, ll &x, ll &y)
{
    if (!b) {x = 1; y = 0; return abs (a);}
    ll d = exgcd (b, (a % b + b) % b, y, x);
    y -= a / b * x;
    return d;
}

```

3.7 CRT

```

ll CRT ()
{
    //ans % a[i] = b[i] if gcd (a[i], a[j]) = 1
    ll sum = 1, ans = 0;
    for (int i = 1; i <= n; ++i) sum *= a[i];
    for (int i = 1; i <= n; ++i)
    {
        ll x, y, tmp = sum / a[i];
        exgcd (tmp, a[i], x, y);
        ans = (ans + tmp * x * b[i]) % sum;
    }
    return (ans + sum) % sum;
}

```

3.8 Lagrange Interpolation

```

ll Lagrange (ll num)
{
    ll ans = 0;
    for (int i = 1; i <= n; ++i)
    {
        ll p = y[i] % MOD, q = 1;
        for (int j = 1; j <= n; ++j)
        {
            if (i == j) continue;
            p = p * (num - x[j]) % MOD;
            q = q * (x[i] - x[j]) % MOD;
        }
        ans = (ans + p * qpow (q, MOD - 2) % MOD + MOD) % MOD;
    }
    return ans;
}

```

3.9 Gaussian Elimination

```

vector <vector <double>> a (n + 1, vector <double> (n + 2)); // n * (n + 1)
for (int i = 1; i <= n; ++i)

```

```

{
    int p = cur, ok = 0;
    while (p <= n)
    {
        if (a[p][i] != 0) {ok = 1; break;}
        ++p;
    }
    if (!ok) continue;
    for (int j = i; j <= n + 1; ++j) swap (a[p][j], a[cur][j]);
    for (int j = n + 1; j >= i; --j) a[cur][j] /= a[cur][i];
    for (int j = 1; j <= n; ++j)
    {
        if (j == cur) continue;
        for (int k = n + 1; k >= i; --k) a[j][k] -= a[cur][k] * a[j][i];
    }
    ++cur;
}
if (cur <= n) puts ("No Solution"); //0 solution or infinite solutions
else {for (int i = 1; i <= n; ++i) printf ("%0.6lf\n", a[i][n + 1]);}

```

3.10 FFT

```

using cd = complex <double>;
class FFT
{
    const double Pi = acos (-1);
    vector <int> rev; int n = 1;
    void fft (vector <cd> &a, int op)
    {
        for (int i = 0; i < n; ++i)
            if (i < rev[i]) swap (a[i], a[rev[i]]);
        for (int len = 1; len < n; len <= 1)
        {
            cd wn (cos (Pi / len), op * sin (Pi / len));
            for (int i = 0; i < n; i += len << 1)
            {
                cd w (1);
                for (int j = 0; j < len; ++j)
                {
                    cd u = a[i + j], v = a[i + j + len] * w;
                    a[i + j] = u + v; a[i + j + len] = u - v;
                    w *= wn;
                }
            }
        }
        if (op == -1) {for (auto &x : a) x /= n;}
    }
public :
    vector <double> conv (vector <cd> a, vector <cd> b)
    {
        int L = 0, tot = (int)a.size () + (int)b.size () - 1; n = 1;
        while (n < tot) n <= 1, ++L;
        rev.resize (n); a.resize (n); b.resize (n);
        for (int i = 0; i < n; ++i) rev[i] = (rev[i] >> 1) >> 1 | ((i & 1) << (L - 1));
        fft (a, 1); fft (b, 1);
        for (int i = 0; i < n; ++i) a[i] = a[i] * b[i];
        fft (a, -1);
        vector <double> ans (tot);
        for (int i = 0; i < tot; ++i) ans[i] = a[i].real ();
        return ans;
    }
};

```

3.11 NTT

```

class NTT
{
    vector<int> rev;int n = 1;
    const int P = 1004535809;//998244353 469762049 500000000001507329LL
        4179340454199820289
    const int G = 3;
    //1945555039024054273 for G = 5
    int qpow (int x,int y)
    {
        int res = 1;
        while (y)
        {
            if (y & 1) res = 1ll * res * x % P;
            x = 1ll * x * x % P;
            y >>= 1;
        }
        return res;
    }
    void ntt (vector<int> &a,int op)
    {
        for (int i = 0;i < n;++i)
            if (i < rev[i]) swap (a[i],a[rev[i]]);
        for (int len = 1;len < n;len <<= 1)
        {
            int wn = qpow (3,(P - 1) / (len << 1));
            if (op == -1) wn = qpow (wn,P - 2);
            for (int i = 0;i < n;i += len << 1)
            {
                int w = 1;
                for(int j = 0;j < len;++j)
                {
                    int u = a[i + j] % P,v = 1ll * a[i + j + len] * w % P;
                    a[i + j] = (0ll + u + v) % P;
                    a[i + j + len] = (0ll + u - v + P) % P;
                    w = 1ll * w * wn % P;
                }
            }
        }
        if (op == -1)
        {
            int inv_n = qpow (n,P - 2);
            for (auto &x : a) x = 1ll * x * inv_n % P;
        }
    }
public :
    vector<int> conv (vector<int> a,vector<int> b)
    {
        int L = 0,tot = (int)a.size () + (int)b.size () - 1;n = 1;
        while (n < tot) n <<= 1,++L;
        rev.resize (n);a.resize (n);b.resize (n);
        for (int i = 0;i < n;++i) rev[i] = (rev[i >> 1] >> 1) | ((i & 1) << (L -
            1));
        ntt (a,1);ntt (b,1);
        for (int i = 0;i < n;++i) a[i] = 1ll * a[i] * b[i] % P;
        ntt (a,-1);a.resize (tot);
        return a;
    }
};

```

3.12 FWT

```

template <typename T>
class FWT
{
public:
const int P = 998244353;
const ll inv_2 = 499122177;//5000000004 for 1e9 + 7
private:
void Or (vector <T> &F,int op = 1)
{
int n = F.size ();
for (int i = 2;i <= n;i <<= 1)
{
int o = i >> 1;
for (int j = 0;j < n;j += i)
for (int k = 0;k < o;++k) F[j + k + o] = (F[j + k + o] + F[j + k]
* op % P) % P;
}
}
void And (vector <T> &F,int op = 1)
{
int n = F.size ();
for (int i = 2;i <= n;i <<= 1)
{
int o = i >> 1;
for (int j = 0;j < n;j += i)
for (int k = 0;k < o;++k) F[j + k] = (F[j + k] + F[j + k + o] *
op % P) % P;
}
}
void Xor (vector <T> &F,int op = 1)
{
int n = F.size ();
for (int i = 2;i <= n;i <<= 1)
{
int o = i >> 1;
for (int j = 0;j < n;j += i)
{
for (int k = 0;k < o;++k)
{
F[j + k] = (F[j + k] + F[j + k + o]) % P;
F[j + k + o] = (F[j + k] - 2 * F[j + k + o] % P + P) % P;
F[j + k] = F[j + k] * op % P;
F[j + k + o] = F[j + k + o] * op % P;
}
}
}
}
vector <T> mul (vector <T> A,vector <T> B)
{
int n = A.size ();
vector <T> C (n);
for (int i = 0;i < n;++i) C[i] = A[i] * B[i] % P;
return C;
}
public :
FWT () {}
vector <T> calc_Or (vector <T> A,vector <T> B)
{
Or (A);Or (B);
auto C = mul (A,B);
Or (C,P - 1);
return C;
}
}

```

```

vector <T> calc_And (vector <T> A,vector <T> B)
{
    And (A);And (B);
    auto C = mul (A,B);
    And (C,P - 1);
    return C;
}
vector <T> calc_Xor (vector <T> A,vector <T> B)
{
    Xor (A);Xor (B);
    auto C = mul (A,B);
    Xor (C,inv_2);
    return C;
}
};

```

4 Computational Geometry

```

using LD = long double;
const LD pi = acos (-1.0);
const LD eps = 1e-8;
int dcmp (LD x) {return x < -eps ? -1 : (x > eps ? 1 : 0);}
struct Point {LD x,y;Point (LD x = 0,LD y = 0) : x (x),y (y) {};}
struct Circle {Point O;LD r;Circle (Point O = Point (),LD r = 0) : O (O),r (r)
{};};
typedef Point Vector;
Vector operator + (Vector A,Vector B) {return Vector (A.x + B.x,A.y + B.y);}
Vector operator - (Vector A,Vector B) {return Vector (A.x - B.x,A.y - B.y);}
Vector operator * (Vector A,LD k) {return Vector (A.x * k,A.y * k);}
Vector operator / (Vector A,LD k) {return Vector (A.x / k,A.y / k);}
bool operator == (Vector A,Vector B) {return dcmp (A.x - B.x) == 0 && dcmp (A.y -
    B.y) == 0;}
bool operator != (Vector A,Vector B) {return !(A == B);}

LD dot (Vector A,Vector B) {return A.x * B.x + A.y * B.y;}
LD dis (Point A,Point B) {return sqrt ((A.x - B.x) * (A.x - B.x) + (A.y - B.y) *
    (A.y - B.y));}
LD cross (Vector A,Vector B) {return A.x * B.y - A.y * B.x;} // A -> B counter-
    clockwise if cross (A,B) > 0
LD len (Point A) {return sqrt (A.x * A.x + A.y * A.y);}
LD angle (Vector A,Vector B) {return acos (dot (A,B) / (len (A) * len (B)));}
Vector proj (Vector A,Vector B) {return A * (dot (A,B) / dot (A,A));} // project
    onto A
Point foot (Point P,Point A,Point B) {Vector AP = P - A,AB = B - A;return A +
    proj (AB,AP);} // foot
Point reflect (Point P,Point A,Point B) {Point F = foot (P,A,B);return F * 2 - P
    ;} // symmetry point
Point rotate (Point P,LD theta) {return (Point){P.x * cos (theta) - P.y * sin (
    theta),P.x * sin (theta) + P.y * cos (theta)};} // counterclockwise
bool chk_on_line (Point P,Point A,Point B) {return dcmp (cross (P - A,B - A)) ==
    0;} // judge whether on line AB
bool chk_on_seg (Point P,Point A,Point B) {return chk_on_line (P,A,B) && dcmp (
    dot (P - A,P - B)) <= 0;} // judge whether on segment AB
LD dis_seg (Point P,Point A,Point B) // the distance from point P to segment AB.
{
    if (dcmp (dot (B - A,P - A)) < 0) return dis (P,A);
    if (dcmp (dot (A - B,P - B)) < 0) return dis (P,B);
    return fabs (cross (P - A,P - B)) / dis (A,B);
}
Point ll_inter_pt (Point A,Point B,Point C,Point D) {return A + (B - A) * cross (
    C - A,D - C) / cross (B - A,D - C);}
bool chk_ll_inter (Point A,Point B,Point C,Point D) {return dcmp (cross (B - A,D
    - C)) != 0;} // line - line

```

```

bool chk_ls_inter (Point A,Point B,Point C,Point D) {return chk_on_line (
    ll_inter_pt (A,B,C,D),C,D);} // The intersection of AB(line) and CD (line) is
    on the CD (seg).
bool chk_ss_inter (Point A,Point B,Point C,Point D) // seg - seg
{
    LD c1 = cross (B - A,C - A),c2 = cross (B - A,D - A);
    LD d1 = cross (D - C,A - C),d2 = cross (D - C,B - C);
    if (dcmp (c1) * dcmp (c2) < 0 && dcmp (d1) * dcmp (d2) < 0) return true;
    if (dcmp(c1) == 0 && chk_on_seg (C,A,B)) return true;
    if (dcmp(c2) == 0 && chk_on_seg (D,A,B)) return true;
    if (dcmp(d1) == 0 && chk_on_seg (A,C,D)) return true;
    if (dcmp(d2) == 0 && chk_on_seg (B,C,D)) return true;
    return false;
}
bool SameSide (Point A,Point B,Point C,Point D) {return cross (A - C,D - C) *
    cross (D - C,B - C) < 0;} // Point A,B Line C,D

LD area (vector <Point> &P)
{
    int n = P.size ();
    LD res = 0;
    for (int i = 0;i < n;++i) res += cross (P[i],P[(i + 1) % n]);
    return res / 2.0;
}
bool is_convex (vector <Point> &P)
{
    int n = P.size ();
    for(int i = 0;i < n - 1;++i)
        if (dcmp (cross (P[i + 1] - P[i],P[(i + 2) % n] - P[i])) < 0) return
            false;
    return true;
}
int in_Poly (vector <Point> &P,Point A) // O (n) for any polygons
{
    int cnt = 0,n = P.size ();
    for (int i = 0;i < n;++i)
    {
        int j = (i + 1) % n;
        if (chk_on_seg (A,P[i],P[j])) return 2;// on the edge
        if (A.y >= min (P[i].y,P[j].y) && A.y < max (P[i].y,P[j].y)) // the
            intersection is on the right
            cnt += dcmp (((A.y - P[i].y) * (P[j].x - P[i].x) / (P[j].y - P[i].y)
                + P[i].x) - A.x) > 0;
    }
    return cnt & 1;
}
int in_convex_Poly (vector <Point> &P,Point A) // O (log n) only for convex
polygons
{
    int n = P.size ();
    if (chk_on_seg (A,P[0],P[1]) || chk_on_seg (A,P[0],P[n - 1])) return 2;
    if (dcmp (cross (P[n - 1] - P[0],A - P[0])) > 0 || dcmp (cross (P[1] - P[0],A
        - P[0])) < 0) return 0;
    int l = 1,r = n - 2,res = -1;
    while (l <= r)
    {
        int mid = (l + r) >> 1;
        if (dcmp (cross (P[mid] - P[0],A - P[0])) >= 0) res = mid,l = mid + 1;
        else r = mid - 1;
    }
    if (chk_on_seg (A,P[res],P[res + 1])) return 2;
    if (dcmp (cross (P[res + 1] - P[res],A - P[res])) < 0) return 0;
    return 1;
}

```

```

}
auto convex_hull (vector <Point> &P) // strict convex hull (<= 0)
{
    int n = P.size ();
    sort (P.begin (),P.end (),[] (Point &x,Point &y) {return x.x == y.x ? x.y < y
        .y : x.x < y.x;});
    vector <Point> hull;
    hull.resize (2 * n + 1);
    int k = 0;
    for (int i = 0;i < n;++i)
    {
        while (k >= 2 && dcmp (cross (hull[k - 1] - hull[k - 2],P[i] - hull[k -
            2])) <= 0) --k;
        hull[k++] = P[i];
    }
    for (int i = n - 2,t = k;i >= 0;--i)
    {
        while (k > t && dcmp (cross (hull[k - 1] - hull[k - 2],P[i] - hull[k -
            2])) <= 0) --k;
        hull[k++] = P[i];
    }
    hull.resize (k - 1);
    return hull;
}
LD diameter (vector <Point> &P)
{
    int n = P.size ();
    if (n <= 1) return 0;
    if (n == 2) return len (P[1] - P[0]);
    LD res = 0;
    for (int i = 0,j = 2;i < n;++i)
    {
        while (dcmp (cross (P[(i + 1) % n] - P[i],P[j] - P[i]) - cross (P[(i + 1)
            % n] - P[i],P[(j + 1) % n] - P[i])) <= 0) j = (j + 1) % n;
        res = max (res,max (len (P[j] - P[i]),len (P[j] - P[(i + 1) % n])));
    }
    return res;
}
template <typename Line = pair <Point, Point>>
LD half_plane (vector <Line> &Vec)
{
    int n = Vec.size ();
    auto get_angle = [&] (Line line) -> LD {return atan2 (line.second.y - line.
        first.y,line.second.x - line.first.x);};
    sort (Vec.begin (),Vec.end (),[&] (Line A,Line B)
    {
        LD angA = get_angle (A),angB = get_angle (B);
        return fabs (angA - angB) > eps ? angA < angB : cross (A.second - A.first
            ,B.second - A.first) < 0;
    });
    int h = 1,t = 0;
    auto check = [&] (Line x,Line y,Line z) -> bool
    {
        Point P = ll_inter_pt (y.first,y.second,z.first,z.second);
        return dcmp (cross (x.second - x.first,P - x.first)) < 0;
    };
    vector <Line> q (n + 10);q[++] = Vec[0];
    for (int i = 1;i < n;++i)
    {
        if (get_angle (Vec[i]) - get_angle (Vec[i - 1]) < eps) continue;
        while (h < t && check (Vec[i],q[t],q[t - 1])) --t;
        while (h < t && check (Vec[i],q[h],q[h + 1])) ++h;
        q[++] = Vec[i];
    }
}

```

```

}
while (h < t && check (q[h],q[t],q[t - 1])) --t;
q[++t] = q[h];
vector <Point> p;
for (int i = h;i < t;++i) p.push_back (ll_inter_pt (q[i].first,q[i].second,q[
    i + 1].first,q[i + 1].second));
return area (p);
}
vector <Point> minkowski (vector <Point> &vecA,vector <Point> &vecB)
{
    int n = vecA.size (),m = vecB.size ();
    vector <Point> A (n),B (m),C {vecA[0] + vecB[0]};
    for (int i = 0;i < n;++i) A[i] = vecA[(i + 1) % n] - vecA[i];
    for (int i = 0;i < m;++i) B[i] = vecB[(i + 1) % m] - vecB[i];
    int posa = 0,posb = 0;
    while (posa < n || posb < m)
    {
        if (posa == n) C.push_back (C.back () + B[posb++]);
        else if (posb == m) C.push_back (C.back () + A[posa++]);
        else if (dcmp (cross (A[posa],B[posb])) >= 0) C.push_back (C.back () + A[
            posa++]);
        else C.push_back (C.back () + B[posb++]);
    }
    return convex_hull (C);
}

bool chk_in_cir (Circle C,Point P) {return dcmp (len (P - C.O) - C.r) <= 0;}
Point get_cir_pt (Circle C,LD theta) {return {C.O.x + C.r * cos (theta),C.O.y + C
    .r * sin (theta)};};
int chk_lc_inter (Point A,Point B,Circle C)
{
    LD d = dis_seg (C.O,A,B);
    if (dcmp (d - C.r) == 0) return 0; // tangent
    if (dcmp (d - C.r) > 0) return -1; // separation
    return 1; // intersection
}
int chk_cc_inter (Circle A,Circle B) // the number of tangent lines
{
    LD d = len (A.O - B.O);
    if (dcmp (A.r + B.r - d) < 0) return 4; // externally separate
    if (dcmp (A.r + B.r - d) == 0) return 3; // externally tangent
    if (dcmp (fabs (A.r - B.r) - d) == 0) return 1; // internally tangent
    if (dcmp (fabs (A.r - B.r) - d) > 0) return 0; // one circle inside the other
    return 2; // intersection
}
pair <Point,Point> lc_inter (Point A,Point B,Circle C)
{
    Point F = foot (C.O,A,B);LD d = dis (C.O,F);
    Vector E = (B - A) / dis (A,B);
    Point P1 = F - E * sqrt (C.r * C.r - d * d);
    Point P2 = F + E * sqrt (C.r * C.r - d * d);
    return {P1,P2};
}
pair <Point,Point> cc_inter (Circle A,Circle B)
{
    Vector k = B.O - A.O;
    LD d = len (k);
    LD alpha = atan2 (k.y,k.x),beta = acos ((A.r * A.r + d * d - B.r * B.r) / (2
        * A.r * d));
    Point P1 = get_cir_pt (A,alpha - beta),P2 = get_cir_pt (A,alpha + beta);
    return {P1,P2};
}
pair <Point,Point> tan_cir (Point P,Circle C)

```

```

{
    LD d = len (C.O - P),theta = asin (C.r / d);
    Vector E = (C.O - P) / d;
    Vector P1 = P + (rotate (E,theta) * sqrt (d * d - C.r * C.r));
    Vector P2 = P + (rotate (E,-theta) * sqrt (d * d - C.r * C.r));
    return {P1,P2};
}
Circle tri_incir (Point A,Point B,Point C)
{
    LD a = dis (B,C),b = dis (A,C),c = dis (A,B);
    Point O = (A * a + B * b + C * c) / (a + b + c);
    return {O,dis_seg (O,A,B)};
}
Circle tri_circum (Point A,Point B,Point C)
{
    LD Bx = B.x - A.x,By = B.y - A.y,Cx = C.x - A.x,Cy = C.y - A.y;
    LD D = 2 * (Bx * Cy - By * Cx);
    LD x = (Cy * (Bx * Bx + By * By) - By * (Cx * Cx + Cy * Cy)) / D + A.x;
    LD y = (Bx * (Cx * Cx + Cy * Cy) - Cx * (Bx * Bx + By * By)) / D + A.y;
    Point P (x,y);
    return Circle (P,dis (A,P));
}
auto get_tangents (Circle A,Circle B)
{
    vector <pair <Point,Point>> tangents;
    LD d = len (A.O - B.O),dif = A.r - B.r,sum = A.r + B.r;
    if (dcmp (d - fabs (dif)) < 0) return tangents;
    LD base = atan2 (B.O.y - A.O.y,B.O.x - A.O.x);
    if (dcmp (d - fabs (dif)) == 0)
    {
        tangents.push_back ({get_cir_pt (A,base + (A.r < B.r ? pi : 0)),
            get_cir_pt (A,base + (A.r < B.r ? pi : 0))});
        return tangents;
    }
    LD theta = acos (dif / d);
    tangents.push_back ({get_cir_pt (A,base + theta),get_cir_pt (B,base + theta)
        });
    tangents.push_back ({get_cir_pt (A,base - theta),get_cir_pt (B,base - theta)
        });
    if (dcmp (d - sum) == 0) tangents.push_back ({get_cir_pt (A,base),get_cir_pt
        (A,base)});
    if (dcmp (d - sum) > 0)
    {
        theta = acos (sum / d);
        tangents.push_back ({get_cir_pt (A,base + theta),get_cir_pt (B,base +
            theta + pi)});
        tangents.push_back ({get_cir_pt (A,base - theta),get_cir_pt (B,base -
            theta + pi)});
    }
    return tangents;
}
LD tri_cir_area (Point A,Point B,Circle C)
{
    Vector OA = A - C.O,OB = B - C.O;
    LD S = cross (OA,OB),sign = dcmp (cross (OA,OB)) > 0 ? 1 : -1;
    bool da = dcmp (len (OA) - C.r) < 0,db = dcmp (len (OB) - C.r) < 0;
    if (dcmp (S) == 0) return 0;
    if (da && db) return S * 0.5; // triangle
    if (!da && !db)
    {
        if (chk_lc_inter (A,B,C) == 1)// arc + triangle + arc
        {
            auto [P1,P2] = lc_inter (A,B,C);

```

```

        Vector OP1 = P1 - C.O, OP2 = P2 - C.O;
        if (dis (A,P1) > dis (A,P2)) swap (P1,P2);
        return cross (OP1,OP2) * 0.5 + sign * 0.5 * C.r * C.r * (angle (OA,
            OP1) + angle (OB,OP2));
    }
    else return sign * 0.5 * C.r * C.r * angle (OA,OB); // arc
}
else // triangle + arc
{
    auto [P1,P2] = lc_inter (A,B,C);
    if (chk_on_seg (P2,A,B)) swap (P1,P2);
    Vector OP1 = P1 - C.O;
    if (dcmp (len (OA) - C.r) < 0) return cross (OA,OP1) * 0.5 + sign * 0.5 *
        C.r * C.r * angle (OP1,OB);
    else return cross (OP1,OB) * 0.5 + sign * 0.5 * C.r * C.r * angle (OP1,OA
    );
}
}
LD cc_area (Circle C1,Circle C2)
{
    int op = chk_cc_inter (C1,C2);
    if (op <= 1) return pi * min (C1.r,C2.r) * min (C1.r,C2.r);
    else if (op == 4) return 0;
    else
    {
        LD d = dis (C1.O,C2.O);
        LD alpha = 2 * acos ((C1.r * C1.r - C2.r * C2.r + d * d) / (2 * C1.r * d)
        );
        LD beta = 2 * acos ((C2.r * C2.r - C1.r * C1.r + d * d) / (2 * C2.r * d))
        ;
        return 0.5 * (C1.r * C1.r * (alpha - sin (alpha)) + C2.r * C2.r * (beta -
            sin (beta)));
    }
}
}

```

5 String

5.1 Hash

```
template <unsigned long long base = 13331>
class Hash
{
    using u64 = unsigned long long;
    vector <u64> pw,hsh;
public:
    Hash (char *s)
    {
        int n = strlen (s);
        pw.assign (n + 1,0),hsh.assign (n + 1,0);
        pw[0] = 1;hsh[0] = 0;
        for (int i = 1;i <= n;++i) pw[i] = pw[i - 1] * base,hsh[i] = hsh[i - 1] *
            base + s[i - 1];
    }
    u64 get (int l,int r) {return hsh[r] - hsh[l - 1] * pw[r - l + 1];}
    u64 link (int l1,int r1,int l2,int r2) {return get (l1,r1) * pw[r2 - l2 + 1]
        + get (l2,r2);}
    bool same (int l1,int r1,int l2,int r2) {return get (l1,r1) == get (l2,r2);}
};
```

5.2 KMP

```
class KMP
{
    vector <int> fail,ans;
    void getfail (char *s) // Match a with b
    {
        int len = strlen (s);
        fail.assign (len + 1,0);
        fail[0] = -1;
        for (int i = 1;i < len;++i)
        {
            int cnt = fail[i - 1];
            while (cnt >= 0 && s[cnt + 1] != s[i]) cnt = fail[cnt];
            if (s[cnt + 1] == s[i]) ++cnt;
            fail[i] = cnt;
        }
    }
public:
    auto get_pos (char *s,char *t)
    {
        getfail (t);
        int lens = strlen (s),lent = strlen (t),cnt = -1;
        for (int i = 0;i < lens;++i)
        {
            while (cnt >= 0 && t[cnt + 1] != s[i]) cnt = fail[cnt];
            if (t[cnt + 1] == s[i])
            {
                ++cnt;
                if (cnt + 1 == lent) ans.push_back (i - lent + 2),cnt = fail[cnt];
            }
        }
        return ans;
    }
};
```

5.3 Manacher

```

int Manacher (char *s)
{
    int n = strlen (s), cnt = 0, r = 0, mid = 0, ans = 0;
    vector <char> a (2 * n + 5); vector <int> p (2 * n + 5);
    a[+cnt] = '!'; a[+cnt] = '#';
    for (int i = 0; i < n; ++i) a[+cnt] = s[i], a[+cnt] = '#';
    a[+cnt] = '~';
    for (int i = 2; i < cnt; ++i)
    {
        if (i <= r) p[i] = min (r - i + 1, p[mid * 2 - i]);
        else p[i] = 1;
        while (a[i - p[i]] == a[i + p[i]]) ++p[i];
        if (i + p[i] > r) r = i + p[i] - 1, mid = i;
        ans = max (ans, p[i]);
    }
    return ans - 1;
}

```

5.4 Trie

5.4.1 Trie

```

struct Trie
{
    int n, m, cnt; //m total len
    vector <vector <int>> ch; vector <int> vis;
    public :
    Trie (int n, int m) : n (n), m (m), ch (m, vector <int> (26, 0)), vis (m) {cnt = 0;}
    void insert (char *s)
    {
        int u = 0, len = strlen (s);
        for (int i = 0; i < len; ++i) m
        {
            int c = s[i] - 'a';
            if (!ch[u][c]) ch[u][c] = ++cnt;
            u = ch[u][c];
        }
        ++vis[u];
    }
    int query (char *s)
    {
        int u = 0, len = strlen (s);
        for (int i = 0; i < len; ++i)
        {
            int c = s[i] - 'a';
            if (!ch[u][c]) return 0;
            u = ch[u][c];
        }
        return vis[u];
    }
};

```

5.4.2 01-Trie

```

class Trie
{
    int n, cnt;
    vector <vector <int>> ch;
    vector <int> val, w;
    public :
    Trie (int n, int lg) : n (n), val (2 * lg * n + 1, 0), w (2 * lg * n + 1, 0), ch (2 * lg * n + 1, vector <int> (2, 0)) {cnt = 1;}
    void pushup (int u)

```

```

{
    w[u] = val[u] = 0;
    //w[u] Number of values (weights) on the edge between node u and its
    parent node
    //val[u] XOR sum maintained by the subtree rooted at u
    if (ch[u][0]) w[u] ^= w[ch[u][0]], val[u] ^= val[ch[u][0]] << 1;
    if (ch[u][1]) w[u] ^= w[ch[u][1]], val[u] ^= (val[ch[u][1]] << 1) | w[ch[u]
    ][1]];
}
void modify (int &u,int v,int dep)
{
    if (!u) u = ++cnt;
    w[u] ^= 1;
    if (dep < 0) return ;
    modify (ch[u][v & 1],v >> 1,dep - 1);
    pushup (u);
}
void erase (int u,int v,int dep)
{
    if (!u) return ;
    w[u] ^= 1;
    if (dep < 0) return ;
    erase (ch[u][v & 1],v >> 1,dep - 1);
    pushup (u);
}
void add (int u) // add 1 in [1,n]
{
    swap (ch[u][0],ch[u][1]);
    if (ch[u][0]) add (ch[u][0]);
    pushup (u);
}
};

```

5.5 Aho-Corasick Automaton

```

class AC
{
    int n,m,cnt;//m total len
    vector <vector <int>> tr;vector <int> val;
public :
    Trie (int n,int m) : n (n),m (m),tr (m + 1,vector <int> (26,0)),val (m + 1),
        fail (m + 1) {cnt = 1;}
    void insert (char *s)
    {
        int n = strlen (s + 1),u = 1;
        for (int i = 1;i <= N;++i)
        {
            if (!tr[u][s[i] - 'a']) tr[u][s[i] - 'a'] = ++cnt;
            u = tr[u][s[i] - 'a'];
        }
        ++val[u];
    }
    void build ()
    {
        for (int i = 0;i < 26;++i) tr[0][i] = 1;
        queue <int> q;q.push (1);
        while (!q.empty ())
        {
            int u = q.front ();q.pop ();
            for (int i = 0;i < 26;++i)
            {
                if (!tr[u][i]) tr[u][i] = tr[fail[u]][i];
                else fail[tr[u][i]] = tr[fail[u]][i],q.push (tr[u][i]);
            }
        }
    }
}

```

```

    }
}
int query (char *s)
{
    int n = strlen (s + 1),u = 1,res = 0;
    for (int i = 1;i <= n;++i)
    {
        u = tr[u][s[i] - 'a'];
        for (int j = u;j && ~val[j];j = fail[j]) res += val[j],val[j] = -1;
    }
    return res;
}
};
/*
s :
a
aa
aaa
t :
aaaa
ans : 4
*/

```

5.6 SAM

```

class SAM
{
    class node
    {
        public:int ch[26],len,fa;
        node (const int &L = 0) {memset (ch,0,sizeof (ch));fa = 0;len = L;}
    };
    public:
    vector <node> t;int lst;
    void GetParentTree (vector <vector <int>> &G)
    {
        G.resize (t.size ());
        for (unsigned i = 1;i < t.size ();++i) G[t[i].fa].push_back (i);
    }
    void extend (const int &c)
    {
        int p = lst,np = lst = t.size ();
        t.push_back (node (t[p].len + 1));
        for (;p&&!t[p].ch[c];p = t[p].fa) t[p].ch[c] = np;
        if (!p) t[np].fa = 1;
        else
        {
            int v = t[p].ch[c];
            if (t[v].len == t[p].len + 1) t[np].fa = v;
            else
            {
                int nv = t.size ();t.push_back (t[v]);
                t[nv].len = t[p].len + 1;
                for (;p && t[p].ch[c] == v;p = t[p].fa) t[p].ch[c] = nv;
                t[np].fa = t[v].fa = nv;
            }
        }
    }
}
SAM () {t.assign (2,node ());lst = 1;}
inline void clear () {t.assign (2,node ());lst = 1;}
inline int next (int p,int c) {return t[p].ch[c];}
inline int Len (int p) {return t[p].len;}
};

```

6 Other

6.1 Fast I/O

```
#include <bits/stdc++.h>
#define init(x) memset (x,0,sizeof (x))
#define INF 0x3f3f3f3f
#define pii pair <int,int>
using i64 = long long;
using u64 = unsigned long long;
using LD = long double;
using namespace std;
const int MAX = 1e5 + 5;
const int MOD = 1e9 + 7;
char s[200];
inline int read ();
inline void output ();
int main ()
{
    //freopen (".in","r",stdin);
    //freopen (".out","w",stdout);
    return 0;
}
inline int read ()
{
    int s = 0;int f = 1;
    char ch = getchar ();
    while ((ch < '0' || ch > '9') && ch != EOF)
    {
        if (ch == '-') f = -1;
        ch = getchar ();
    }
    while (ch >= '0' && ch <= '9')
    {
        s = s * 10 + ch - '0';
        ch = getchar ();
    }
    return s * f;
}
inline void output (int x)
{
    if (x < 0) putchar ('-');
    x = (x > 0) ? x : -x;
    int cnt = 0;
    while (x)
    {
        s[cnt++] = x % 10 + '0';
        x /= 10;
    }
    while (cnt) putchar (s[--cnt]);
}
}
```

6.2 Execution

```
#!/bin/bash
ulimit -s 102400
g++ -std=c++17 -O2 -Wall "$1" -o main
./main < in.txt > out.txt

# Set-ExecutionPolicy -Scope Process -ExecutionPolicy Bypass
# new file : 'type nul > filename.cpp'
# run : '.\run.ps1 filename.cpp'
g++ -std=c++17 -O2 -Wall $args[0] -o main
```

```
cat in.txt | .\main | Out-File -FilePath out.txt
```

6.3 Comparison

```
#!/bin/bash
for ((i=1; ; i++)); do
    ./gen > input.txt # generator
    ./code < input.txt > output1.txt # your code
    ./std < input.txt > output2.txt # std code
    if diff output1.txt output2.txt; then # compare
        echo "Test $i: Accepted"
    else
        echo "Test $i: Wrong Answer"
        echo "Input:"
        cat input.txt
        echo "Your Output:"
        cat output1.txt
        echo "Standard Output:"
        cat output2.txt
        break
    fi
done
```

6.4 Python

```
import sys
sys.set_int_max_str_digits (value) # set big number
sys.setrecursionlimit (deep) # set deeper recursion
lst = sorted (lst,key = lambda x : x[1]) # sort by the second element
```